9.1 Sequences + Series (Part 2)

Fibonacci Sequences

ex

$a_0 = \boxed{1}$

$a_1 = \boxed{1}$

$a_2 = a_{n-2} + a_{n-1}$

$= a_{2-2} + a_{2-1}$

$= a_0 + a_1 = 1 + 1 = \boxed{2}$

$a_3 = a_{n-2} + a_{n-1}$

$= a_{3-2} + a_{3-1}$

$= a_1 + a_2$

$= 1 + 2 = \boxed{3}$

See the pattern add last 2 terms

$a_4 = 2 + 3 = \boxed{5}$

$a_5 = 3 + 5 = \boxed{8}$

1st term

Find 1st 5 terms

(52) $a_1 = 3$

$a_{k+1} = 2(a_k - 1)$

$a_2 = a_{1+1} = 2(a_1 - 1)$

k

$= 2(3-1) = 4$

$a_3 = a_{2+1} = 2(a_2 - 1)$

$2(4-1) = 6$

$a_4 = 2(6-1) = 10$

$a_5 = 2(10-1) = 18$

$\boxed{3, 4, 6, 10, 18}$

(55) $a_0 = 1$

$a_1 = 2$

$a_2 = a_k = a_{k-2} + \frac{1}{2}a_{k-1}$

$= 1 + \frac{1}{2}(2) = 2$

$a_3 = 2 + \frac{1}{2}(2) = 3$

$a_4 = 2 + \frac{1}{2}(3) = 3\frac{1}{2}$

$\boxed{1, 2, 2, 3, 3\frac{1}{2}}$

Factorials

$n! = n \cdot (n-1) \cdot (n-2) \cdot \ldots \cdot 1$

$5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$

Need to know $\boxed{0! = 1}$

(59) $a_n = \frac{5}{n!}$

$a_0 = \frac{5}{0!} = \frac{5}{1} = \boxed{5}$

$a_1 = \frac{5}{1!} = \frac{5}{1} = \boxed{5}$

$a_2 = \frac{5}{2!} = \frac{5}{2 \cdot 1} = \boxed{\frac{5}{2}}$

$a_3 = \frac{5}{3!} = \frac{5}{3 \cdot 2 \cdot 1} = \boxed{\frac{5}{6}}$

$a_4 = \frac{5}{4!} = \frac{5}{4 \cdot 3 \cdot 2 \cdot 1} = \boxed{\frac{5}{24}}$

ex $\frac{8!}{2!6!} = \frac{8 \cdot 7 \cdot \cancel{6!}}{2 \cdot 1 \cdot \cancel{6!}} = \frac{8 \cdot 7}{2 \cdot 1} = \frac{56}{2} = \boxed{28}$

66 $\frac{(2n-1)!}{(2n+1)!} = \frac{\cancel{(2n-1)!}}{(2n+1)(2n)\cancel{(2n-1)!}} = \boxed{\frac{1}{2n(2n+1)}}$

Summation

Sigma Symbol

$\sum$ means add each term

$$\boxed{\sum_{i=1}^{n} a_i = a_1 + a_2 + a_3 + \dots a_n}$$

$\sum_{i=1}^{n} c = c(n)$ → c is a constant

Can also be moved to the front $\sum_{i=1}^{n} c\,a_n$

$c \sum_{i=1}^{n} a_n$

If 2 terms

$$\sum_{i=1}^{n} (a_n + b_n) = \sum_{i=1}^{n} a_n + \sum_{i=1}^{n} b_n$$

67 $\sum_{i=0}^{4} 3i^2 \Rightarrow 3\sum_{i=0}^{4} i^2 \Rightarrow 3(0^2 + 1^2 + 2^2 + 3^2 + 4^2)$

$3(0 + 1 + 4 + 9 + 16) = \boxed{90}$

70 $\sum_{i=1}^{5} 2n-1 \Rightarrow 2\sum_{i=1}^{5} n - \sum_{i=1}^{5} 1$ ← constant so $C(n)$

$2(1+2+3+4+5) - 1(5) = \boxed{25}$

80 write the notation

$\frac{5}{1+1} + \frac{5}{1+2} + \frac{5}{1+3} + \dots + \frac{5}{1+15} = \boxed{\sum_{n=1}^{15} \frac{5}{1+n}}$

changing so n start with 1 and ends with 15

Do Not have to use n as your variable

Find Partial Sum

91) $\sum_{n=1}^{\infty} 4\left(-\frac{1}{2}\right)^n$

3rd) $\underbrace{4\left(-\frac{1}{2}\right)^1 + 4\left(-\frac{1}{2}\right)^2 + 4\left(-\frac{1}{2}\right)^3}_{\text{First 3 terms}} = -\frac{3}{2}$

4th) First 3 terms + 4th

$-3/2 + 4\left(-\frac{1}{2}\right)^4 = -\frac{5}{4}$

5th) $-5/4 + 4\left(-1/2\right)^5 = \frac{-11}{8}$

93) Find sum of infinite series

$$\sum_{i=1}^{\infty} = \frac{6}{10^i} = \frac{6}{10^1} + \frac{6}{10^2} + \frac{6}{10^3} + \frac{6}{10^4} + \cdots$$

$$= .6 + .06 + .006 + .0006 + \cdots$$

$$= .6666\ldots$$

$$= \boxed{2/3}$$

Go to calcview.com to watch videos of problems in red. Can as use camera on phone, put over QR code and it will link to the page on safari

Assignment P. 618 (51-55 odd, 59-83 odd, 90, 94)