8.1 Practice Problems Adapted from AP Classroom (non secure).

1. The average value of a function f over the interval $[-1,2]$ is -4, and the average value of f over the interval $[2,7]$ is 8. What is the average value of f over the interval $[-1,7]$?

2. The average value of a function f over the interval $[-2,3]$ is -6, and the average value of f over the interval $[3,5]$ is 20. What is the average value of f over the interval $[-2,5]$?

3. Find the average value of f on the interval $-4 \leq x \leq 4$? $f(x) = \sqrt{16 - x^2}$.

4. A store is having an all-day sale. The total number of shoppers who have entered the store t hours after the store opens is modeled by the function S defined by $S(t) = 0.5t^4 - 16t^3 + 144t^2$ for $0 \leq t \leq 12$. At time $t = 0$, when the store opens, there are no shoppers in the store. Find the value of $\frac{1}{3} \int_6^9 S'(t)dt$. Using correct units, explain the meaning of $\frac{1}{3} \int_6^9 S'(t)dt$ in the context of this problem.

5. If the average value of a continuous function f on the interval $[-1, 11]$ is 5, what is $\int_{-1}^{11} \frac{f(x)}{6} dx$?

6. Let f be the function that is defined for all real numbers x and that has the following properties.

(i) $f''(x) = 24x - 18$

(ii) $f'(1) = -6$

(iii) $f(2) = 0$

Find the average value of f on the interval $1 \leq x \leq 3$.

7. Grass clippings are in a bin where they decompose. For $0 \le t \le 10$, the amount of grass clippings remaining in the bin is modeled by $A(t) = 7(0.9)^t$, where $A(t)$ is measured in pounds and t is measured in days.

Find the time t for which the amount of grass clippings in the bin is equal to the average amount of grass clippings in the bin over the interval $0 \le t \le 10$.

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8. The average value of $y = \dfrac{1}{x}$ on the closed interval $[1, 3]$ is ?

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9. What is the average value of $y = x^2\sqrt{x^3 + 1}$ on the interval $[0, 2]$?

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10. The average value of $y = \sqrt{x}$ over the interval $0 \le x \le 2$ is ?

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11. What is the average (mean) value of $y = 3t^3 - t^2$ over the interval $-1 \le t \le 2$?

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12. What is the average value of $y = \dfrac{x}{x^2+2}$ on the interval $0 \le x \le \sqrt{6}$.

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13. Let f be the function defined by $f(x) = \begin{cases} \sqrt{x+1} & \text{for } 0 \le x \le 3 \\ 5 - x & \text{for } 3 < x \le 5 \end{cases}$.

Find the average value of $f(x)$ on the closed interval $0 \le x \le 5$.

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14. What is the average value for the portion of the curve $y = 3x - x^2$, which is in Quadrant I of the coordinate plane?

Answers:

1. $\dfrac{7}{2}$

2. $\dfrac{10}{7}$

3. π

4. $\dfrac{1}{3}\int_6^9 S'(t)dt = 301.5$; $\dfrac{1}{3}\int_6^9 S'(t)dt$ is the average rate at which the shoppers are entering the store in shoppers/hr between times $t = 6$ and $t = 9$ hours.

5. 10

6. 5

7. The average value, 4.327 pounds, occurs at $t = 4.565$ days.

8. $\dfrac{\ln 3}{2}$

9. $\dfrac{26}{9}$

10. $\dfrac{2}{3}\sqrt{2}$

11. $\dfrac{11}{4}$

12. $\dfrac{1}{\sqrt{6}}\ln 2$

13. $\dfrac{4}{3}$

14. $\dfrac{3}{2}$