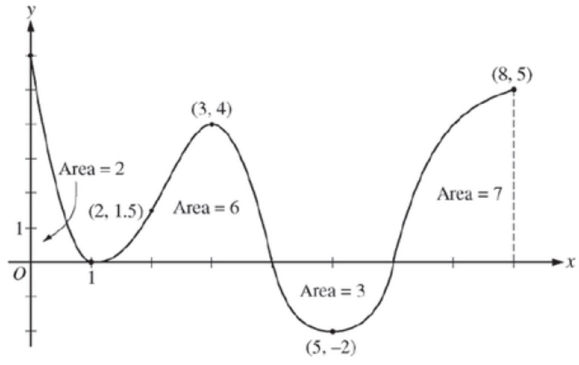
8.3 Practice Adapted from AP Classroom (non-calculator/non-secure)

1.



Graph of f'

The figure above shows the graph of f', the derivative of a twice-differentiable function f, on the closed interval $0 \leq x \leq 8$. The graph of f' has horizontal tangent lines at $x = 1,\ x = 3,$ and $x = 5$. The areas of the regions between the graph of f' and the x-axis are labeled in the figure. The function f is defined for all real numbers and satisfies $f(8) = 4$.

Determine the absolute minimum value of f on the closed interval $0 \leq x \leq 8$. Justify your answer.

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2. For $0 \leq t \leq 6$ seconds, a screen saver on a computer screen shows two circles that start as dots and expand outward.

(a) At the instant that the first circle has a radius of 9 centimeters, the radius is increasing at a rate of $\frac{3}{2}$ cm/s. Find the rate at which the area of the circle is changing at that instant. Indicate units of measure.

(b) The radius of the first circle is modeled by $w(t) = 12 - 12e^{-0.5t}$ for $0 \leq t \leq 6$, where $w(t)$ is measured in centimeters and t is measured in seconds. At what value of time t is the radius of the circle increasing at a rate of 3 cm/s?

(c) A model for the radius of the second circle is given by the function f for $0 \leq t \leq 6$, where $f(t)$ is measured in centimeters and t is measured in seconds. The rate of change of the radius of the second circle is given by $f'(t) = t^2 - 4t + 4$. Based on this model, by how many centimeters does the radius of the second circle increase from time $t = 0$ to $t = 3$?

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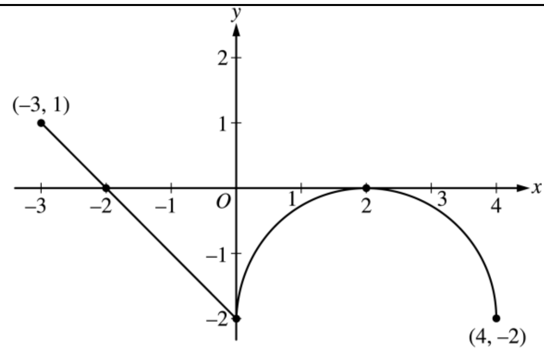
3. Snow is falling at a rate of $r(t) = 2e^{-0.1t}$ inches per hour, where t is the time in hours since the beginning of the snowfall. Which of the following expressions gives the amount of snow, in inches, that falls from time $t = 0$ to time $t = 5$ hours?

(A) $2e^{-0.5} - 2$ (B) $0.2 - 0.2e^{-0.5}$ (C) $4 - 4e^{-0.5}$ (D) $20 - 20e^{-0.5}$

4.



Graph of f'

Let f be a function defined on the closed interval $-3 \leq x \leq 4$ with $f(0) = 3$. The graph of f', the derivative of f, consists of one line segment and a semicircle, as shown above. Find $f(-3)$ and $f(4)$. Show the work that leads to your answers.

5.

t (minutes)	0	2	5	7	11	12
$r'(t)$ (feet per minute)	5.7	4.0	2.0	1.2	0.6	0.5

The volume of a spherical balloon expands as the air inside the balloon is heated. The radius of the balloon, in feet, is modeled by a twice-differentiable function $r(t)$, where t is measured in minutes. Selected values of t on the interval $0 \leq t \leq 12$ and corresponding values of $r'(t)$ are shown in the table above. The radius of the balloon is 30 feet when $t = 5$. (Note: The volume of a sphere of radius r is given by $V = \frac{4}{3}\pi r^3$)

Use a right Riemann sum with the five subintervals indicated by the data in the table to approximate $\int_0^{12} r'(t)dt$. Using correct units, explain the meaning of $\int_0^{12} r'(t)dt$ in terms of the radius.

6. A function $f(t)$ gives the rate of evaporation of water, in liters per hour, from a pond, where t is measured in hours since 12 noon. Which of the following gives the meaning of $\int_4^{10} f(t)dt$?

(A)The total volume of water, in liters, that evaporated from the pond during the first 10 hours after 12 noon.

(B) The total volume of water, in liters, that evaporated from the pond between 4 P.M. and 10 P.M.

(C) The net change in the rate of evaporation, in liters per hour, from the pond between 4 P.M. and 10 P.M.

(D) The average rate of evaporation, in liters per hour, from the pond between 4 P.M. and 10 P.M.

(E) The average rate of change in the rate of evaporation, in liters per hour per hour, from the pond between 4 P.M. and 10 P.M.

7. Water is pumped into an underground tank at a constant rate of 8 gallons per minute. Water leaks out of the tank at the rate of $\sqrt{t+1}$ gallons per minute, for $0 \le t \le 120$ minutes. At time $t = 0$, the tank contains 30 gallons of water. How many gallons of water are in the tank at time $t = 3$ minutes?

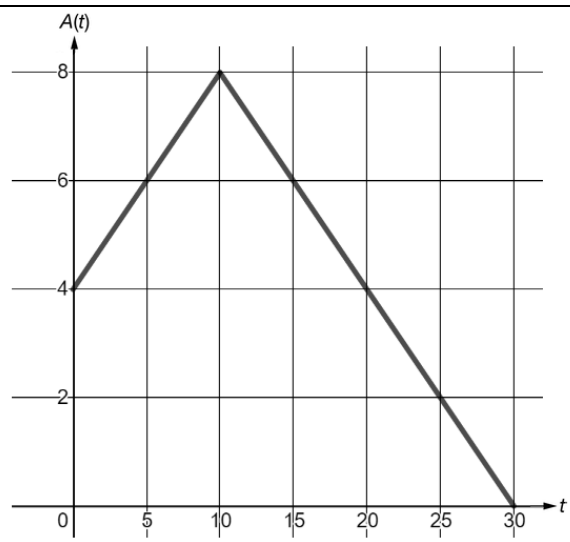
8.

t (days)	0	10	22	30
$W'(t)$ (GL per day)	0.6	0.7	1.0	0.5

The twice-differentiable function W models the volume of water in a reservoir at time t, where $W(t)$ is measured in (GL) and t is measured in days. The table above gives values of $W'(t)$ sampled at various times during the time interval $0 \le t \le 30$ days. At time $t = 30$, the reservoir contains 125 gigaliters of water.

Use a left Riemann sum, with the three subintervals indicated by the data in the table, to approximate $\int_0^{30} W'(t)dt$. Use this approximation to estimate the volume of water $W(t)$, in gigaliters, in the reservoir at time $t = 0$. Show the computations that lead to your answer.

9.



The rate at which ants arrive at a picnic is modeled by the function A, where $A(t)$ is measured in ants per minute and t is measured in minutes. The graph of A for $0 \le t \le 30$ is shown in the figure above. How many ants arrive at the picnic during the time interval $0 \le t \le 30$?

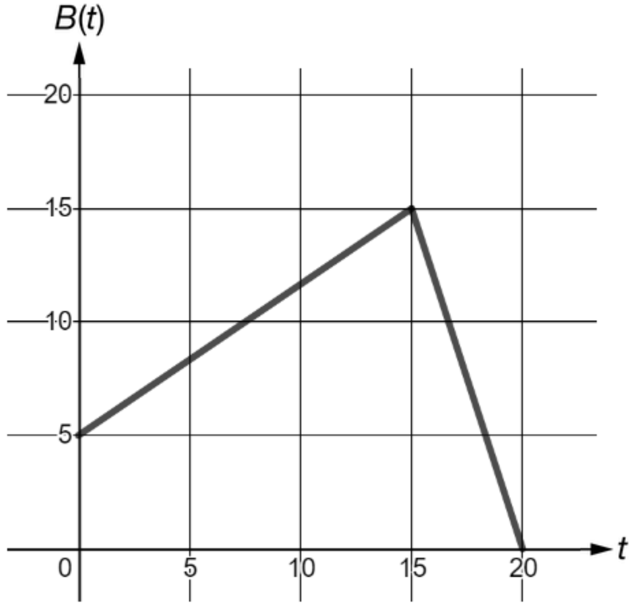
(A) 8

(B) 70

(C) 120

(D) 140

10.



The rate at which people arrive at a theater box office is modeled by the function B, where $B(t)$ is measured in people per minute and t is measured in minutes. The graph of B for $0 \leq t \leq 20$ is shown in the figure above. Which of the following is closest to the number of people that arrive at the box office during the time interval $0 \leq t \leq 20$?

(A) 188

(B) 150

(C) 38

(D) 15

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ANSWERS

1. Critical numbers and enpdoints: $f(0) = -8$, $f(4) = 0$, $f(6) = -3$, $f(8) = 4$. Minimum is -8.

2. (a) When the radius is 9 cm, the area is changing at a rate of 27π cm^3/sec. (b) The radius is increasing at a rate of 3 centimeters per second at time $t = 2\ln 2$ seconds. (c) The radius increases by 3 cm from $t = 0$ to time $t = 3$ seconds.

3. D

4. $f(4) = -5 + 2\pi$, $f(-3) = \frac{9}{2}$

5. $\int_0^{12} r'(t)dt \approx 2(4.0) + 3(2.0) + 2(1.2) + 4(0.6) + 1(0.5) = 19.3$ feet. $\int_0^{12} r'(t)dt$ is the change in the radius in feet from $t = 0$ to $t = 12$ minutes.

6. B

7. Originally: 30 gallons. Pumped in over the first three minutes: 8 (3) = 24 gallons. Leaked out over the first three minutes: $\int_0^3 \sqrt{t+1}\,dt = \frac{14}{3}$. At time $t = 3$ minutes, $30 + 24 - \frac{14}{3} = \frac{148}{3} = 49\frac{1}{3}$ gallons are in the tank.

8. $\int_0^{30} W'(t)dt = 10(0.6) + 12(0.7) + 8(1.0) = 22.4$.
$W(0) = W(30) - \int_0^{30} W'(t)dt = 125 - 22.4 = 102.6$ GL.

9. D

10. A