Contextual Applications of Differentiation — UNIT 4 REVIEW — AP Calc April 16, 2020

4.1 The function $C(x)$ gives the cost of digging a hole x feet deep.

$\boldsymbol{C(20) = 140}$ means that a hole ___20 feet______ deep costs ____$140____ to dig.

$\boldsymbol{C'(20) = 5}$ means that when the hole is ______20 feet deep______, the cost of digging is _increasing_____ at a rate of ____$5 per foot_______________.

4.2 A particle moves back and forth on a horizontal track for $0 \le t < \frac{\pi}{2}$ minutes. The particle's position, in feet, is given by the function $s(t) = \frac{1}{2}\tan t$. Find the acceleration of the particle at time $t = \frac{\pi}{6}$ minutes and indicate units of measure.

$$v(t) = s'(t) = \frac{1}{2}\sec^2 t = \frac{1}{2}(\sec t)^2$$

$$a(t) = v'(t)' = s''(t) = 2 \cdot \frac{1}{2}(\sec t)\sec t \tan t$$

$$a\left(\frac{\pi}{6}\right) = \sec^2\frac{\pi}{6}\tan\frac{\pi}{6} = \left(\frac{2}{\sqrt{3}}\right)^2\left(\frac{\sqrt{3}}{3}\right) = \frac{4}{3\sqrt{3}} = \frac{4\sqrt{3}}{9} \text{ feet per min per min (ft/min}^2\text{)}$$

4.3 If $P(t)$ models the size of a population at time $t > 0$, which of the following differential equations describes linear growth in the size of the population? Which describes exponential growth?

$\frac{dP}{dt} = 200$ Linear Growth	$\frac{dP}{dt} = 200t$	$\frac{dP}{dt} = 100t^2$	$\frac{dP}{dt} = 200P$ Exponential Growth	$\frac{dP}{dt} = 100P^2$

4.4 Determine $\frac{dz}{dt}$ if you know that $z = xy^2$, $z = 3$, $y = \frac{1}{2}$, $\frac{dx}{dt} = -2$, and $\frac{dy}{dt} = 5$.

$$\frac{dz}{dt} = y^2\frac{dx}{dt} + 2xy\frac{dy}{dt}$$

$$3 = x\left(\frac{1}{2}\right)^2 \qquad \left.\frac{dz}{dt}\right|_{\left(12,\frac{1}{2},3\right)} = \left(\frac{1}{2}\right)^2(-2) + 2(12)\left(\frac{1}{2}\right)(5)$$

$$3 = \frac{x}{4} \qquad = -\frac{1}{2} + 60$$

$$x = 12 \qquad = 59\frac{1}{2}$$

4.5 Free Response Question (FRQ) Practice in a bit!!

4.6 Given $g(x)$ is a differentiable function about which little else is known other that $g(-3) = 2$ and $g'(-3) = 7$. Use the tangent line of $g(x)$ at $x = -3$ to approximate $g(-2.9)$.

$$y = 2 + 7(x + 3) \qquad g(-2.9) \approx 2 + 7(-2.9 + 3) = 2 + 0.7 = 2.7$$

4.7 $\lim\limits_{x\to5}\frac{x^4-625}{x^2-25}$

$$\lim_{x\to5}(x^4 - 625) = 0 = \lim_{x\to5}(x^2 - 25)$$

$$\therefore \lim_{x\to5}\frac{x^4 - 625}{x^2 - 25} = \lim_{x\to5}\frac{4x^3}{2x} = \lim_{x\to5}(2x^2) = 2(5)^2 = 50$$

– *OR* –

$$\lim_{x\to5}\frac{(x^2+25(x^2-25)}{x^2-25}$$

$$\lim_{x\to5}(x^2 + 25) = 50$$