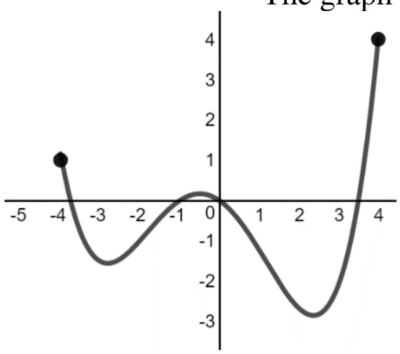
Example 1: Extreme Values / Exploring Accumulations of Change

The graph of a differentiable function $h(x)$ is shown below on the closed interval $[-4, 4]$.



The function $h(x)$ has x-intercepts on the interval $[-4, 4]$ at $x = -3.8$, $x = -1, x = 0$, and $x = 3.4$. Also, $h(x)$ has horizontal tangents at $x = -2.9, x = -0.5$, and $x = 2.2$. The areas of the regions bounded by the x-axis and the graph of $h(x)$ on the intervals $[-1, 0]$, $[0, 3.4]$ and $[3.4, 4]$ are 0.3, 12, and 2, respectively.

For all x, $g(x) = 11 + \int_0^x h(t)dt$.

(a) Find: $g(3.4)$ $g(4)$ $g(-1)$ $g'(3.4)$ $g''(2.2)$

(b) Find the maximum and minimum of $g(x)$ on the interval $[-1, 4]$. Explain your answers.

Example 2: Approximating areas with Riemann Sums

Little is known about the function $v(t)$ except selected values given in the table. Use a ______________ sum with the four sub intervals indicated by the data in the table to approximate $\int_{-2}^{10} v(t)dt$.

t	-2	1	6	7	10
$v(t)$	3	4	-9	2	0

(a) left Riemann

(b) right Riemann

(c) trapezoidal

Note: We did an example of a midpoint sum in one of the Unit 8 videos. ☺

Example 3: Fundamental Theorem of Calculus

(a) $\int_4^9 \frac{1}{2\sqrt{x}}dx =$

(b) $\frac{d}{dx}\left[\int_a^{x^3} \frac{1}{t^2+t-5}dt\right] =$

Example 4: Properties of Definite Integrals

Given $\int_3^6 f(x)dx = 7.5,\quad \int_5^3 f(x)dx = 2,\quad$ and $\quad \int_6^3 g(x)dx = -12,$ determine:

(a) $\int_5^6 f(x)dx$

(b) $\int_3^6 g(x)dx$

(c) $\int_3^6 [4f(x) - 2g(x) + 4]dx$

Example 5: Integration Blast! (Separate paper probably required!)

(a) $\int sin\,\theta d\theta$

(b) $\int \frac{1-x}{1+x^2}dx$

(c) $\int \frac{1}{(2-G)^{2/3}}dG$

(d) $\int_{-5}^{5} \sqrt{25-x^2}dx$

Remember: What's the integral of $\frac{1}{cabin}$ with respect to $cabin$? House Boat!