Adapted from AP Central (with permission)

1. At the beginning of 2010, a landfill contained 1400 tons of solid waste. The increasing function W models the total amount of solid waste stored at the landfill. Planners estimate that W will satisfy the differential equation $\frac{dW}{dt} = \frac{1}{25}(W - 300)$ for the next 20 years. W is measured in tons, and t is measured in years from the start of 2010.

   (a) Use the line tangent to the graph of W at $t = 0$ to approximate the amount of solid waste that the landfill contains at the end of the first 3 months of 2010 (time $t = \frac{1}{4}$).

   (b) Find $\frac{d^2W}{dt^2}$ in terms of W. Use $\frac{d^2W}{dt^2}$ to determine whether your answer in part (a) is an underestimate or an overestimate of the amount of solid waste that the landfill contains at time $t = \frac{1}{4}$.

   (c) Find the particular solution $W = W(t)$ to the differential equation $\frac{dW}{dt} = \frac{1}{25}(W - 300)$ with initial condition $W(0) = 1400$.

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2. Two particles move along the x-axis. For $0 \le t \le 6$, the position of particle P at time t is given by $p(t) = 2\cos\left(\frac{\pi}{4}t\right)$, while the position of particle R at time t is given by $r(t) = t^3 - 6t^2 + 9t + 3$.

   (a) For $0 \le t \le 6$, find all times t during which particle R is moving to the right.

   (b) For $0 \le t \le 6$, find all times t during which the two particles travel in opposite directions.

   (c) Find the acceleration of particle P at time $t = 3$. Is particle P speeding up, slowing down, or doing neither at time $t = 3$? Explain your reasoning.

   (d) There is a third particle Q that moves along the y-axis, perpendicular to the path of particles P and R. The position of particle Q at time t is given by $q(t)$. At time $t = 2$, $q(2) = 4$ and $q'(2) = 6$. Describe how the distance between particles Q and R is changing with respect to time at $t = 2$.

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3. The function f is defined by $f(x) = \sqrt{25 - x^2}$ for $-5 \le x \le 5$.

   (a) Find $f'(x)$.

   (b) Write an equation for the line tangent to the graph of f at $x = -3$.

   (c) Let g be the function defined by $g(x) = \begin{cases} f(x) & \text{for } -5 \le x \le -3 \\ x + 7 & \text{for } -3 < x \le 5. \end{cases}$

   Is g continuous at $x = -3$? Use the definition of continuity to explain your answer.

   (d) Find the value of $\int_0^5 x\sqrt{25 - x^2}\, dx$.

   (e) Find the value of $\int_{-5}^{5} (6f(x) + 3)dx$.