9.3 Exercises

See CalcChat.com for tutorial help and worked-out solutions to odd-numbered exercises.

Vocabulary: Fill in the blanks.

1. A sequence is ________ when the ratios of consecutive terms are the same. This ratio is the ________ ratio.
2. The term of a geometric sequence has the form $a_n =$ ________.
3. The sum of a finite geometric sequence with common ratio $r \neq 1$ is given by $S_n =$ ________.
4. The sum of the terms of an infinite geometric sequence is called a ________ ________.

Skills and Applications

Determining Whether a Sequence Is Geometric In Exercises 5–12, determine whether the sequence is geometric. If so, find the common ratio.

5. $3, 6, 12, 24, \ldots$
6. $5, 10, 15, 20, \ldots$
7. $\frac{1}{27}, \frac{1}{9}, \frac{1}{3}, 1, \ldots$
8. $27, -9, 3, -1, \ldots$
9. $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots$
10. $5, 1, 0.2, 0.04, \ldots$
11. $1, -\sqrt{7}, 7, -7\sqrt{7}, \ldots$
12. $2, \frac{4}{\sqrt{3}}, \frac{8}{3}, \frac{16}{3\sqrt{3}}, \ldots$

Writing the Terms of a Geometric Sequence In Exercises 13–22, write the first five terms of the geometric sequence.

13. $a_1 = 4, r - 3$
14. $a_1 = 7, r = 4$
15. $a_1 = 1, r = \frac{1}{2}$
16. $a_1 = 6, r = -\frac{1}{4}$
17. $a_1 = 1, r = e$
18. $a_1 = 2, r = \pi$
19. $a_1 = 3, r = \sqrt{5}$
20. $a_1 = 4, r = -1/\sqrt{2}$
21. $a_1 = 2, r = 3x$
22. $a_1 = 4, r = x/5$

Finding a Term of a Geometric Sequence In Exercises 23–32, write an expression for the nth term of the geometric sequence. Then find the missing term.

23. $a_1 = 4, r = \frac{1}{2}, a_{10} = \square$
24. $a_1 = 5, r = \frac{7}{2}, a_8 = \square$
25. $a_1 = 6, r = -\frac{1}{3}, a_{12} = \square$
26. $a_1 = 64, r = -\frac{1}{4}, a_{10} = \square$
27. $a_1 = 100, r = e^x, a_9 = \square$
28. $a_1 = 1, r = e^{-x}, a_4 = \square$
29. $a_1 = 1, r = \sqrt{2}, a_{12} = \square$
30. $a_1 = 1, r = \sqrt{3}, a_8 = \square$
31. $a_1 = 500, r = 1.02, a_{40} = \square$
32. $a_1 = 1000, r = 1.005, a_{60} = \square$

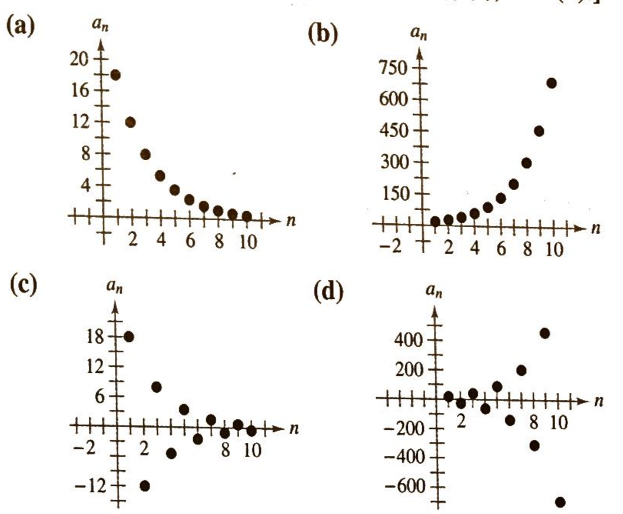
Writing the nth Term of a Geometric Sequence In Exercises 33–38, find a formula for the nth term of the sequence.

33. $64, 32, 16, \ldots$
34. $81, 27, 9, \ldots$
35. $9, 18, 36, \ldots$
36. $5, -10, 20, \ldots$
37. $6, -9, \frac{27}{2}, \ldots$
38. $80, -40, 20, \ldots$

Finding a Term of a Geometric Sequence In Exercises 39–46, find the specified term of the geometric sequence.

39. 8th term: $6, 18, 54, \ldots$
40. 7th term: $5, 20, 80, \ldots$
41. 9th term: $\frac{1}{3}, -\frac{1}{6}, \frac{1}{12}, \ldots$
42. 8th term: $\frac{3}{2}, -1, \frac{2}{3}, \ldots$
43. a_3: $a_1 = 16, a_4 = \frac{27}{4}$
44. a_1: $a_2 = 3, a_5 = \frac{3}{64}$
45. a_6: $a_4 = -18, a_7 = \frac{2}{3}$
46. a_5: $a_2 = 2, a_3 = -\sqrt{2}$

Matching a Geometric Sequence with Its Graph In Exercises 47–50, match the geometric sequence with its graph. [The graphs are labeled (a), (b), (c), and (d).]



47. $a_n = 18\left(\frac{2}{3}\right)^{n-1}$
48. $a_n = 18\left(-\frac{2}{3}\right)^{n-1}$
49. $a_n = 18\left(\frac{3}{2}\right)^{n-1}$
50. $a_n = 18\left(-\frac{3}{2}\right)^{n-1}$

Graphing the Terms of a Sequence In Exercises 51–54, use a graphing utility to graph the first 10 terms of the sequence.

51. $a_n = 14(1.4)^{n-1}$
52. $a_n = 18(0.7)^{n-1}$
53. $a_n = 8(-0.3)^{n-1}$
54. $a_n = 11(-1.9)^{n-1}$

Sum of a Finite Geometric Sequence

In Exercises 55–64, find the sum of the finite geometric sequence.

55. $\sum_{n=1}^{7} 4^{n-1}$

56. $\sum_{n=1}^{10} \left(\frac{3}{2}\right)^{n-1}$

57. $\sum_{n=1}^{6} (-7)^{n-1}$

58. $\sum_{n=1}^{8} 5\left(-\frac{5}{2}\right)^{n-1}$

59. $\sum_{n=0}^{20} 3\left(\frac{3}{2}\right)^{n}$

60. $\sum_{n=0}^{40} 5\left(\frac{3}{5}\right)^{n}$

61. $\sum_{n=0}^{5} 200(1.05)^{n}$

62. $\sum_{n=0}^{6} 500(1.04)^{n}$

63. $\sum_{n=0}^{40} 2\left(-\frac{1}{4}\right)^{n}$

64. $\sum_{n=0}^{50} 10\left(\frac{2}{3}\right)^{n-1}$

Using Summation Notation

In Exercises 65–68, use summation notation to write the sum.

65. $10 + 30 + 90 + \cdot \cdot \cdot + 7290$

66. $15 - 3 + \frac{3}{5} - \cdot \cdot \cdot - \frac{3}{625}$

67. $0.1 + 0.4 + 1.6 + \cdot \cdot \cdot + 102.4$

68. $32 + 24 + 18 + 13.5 + 10.125$

Sum of an Infinite Geometric Series

In Exercises 69–78, find the sum of the infinite geometric series.

69. $\sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^{n}$

70. $\sum_{n=0}^{\infty} 2\left(\frac{3}{4}\right)^{n}$

71. $\sum_{n=0}^{\infty} \left(-\frac{1}{2}\right)^{n}$

72. $\sum_{n=0}^{\infty} 2\left(-\frac{2}{3}\right)^{n}$

73. $\sum_{n=0}^{\infty} (0.8)^{n}$

74. $\sum_{n=0}^{\infty} 4(0.2)^{n}$

75. $8 + 6 + \frac{9}{2} + \frac{27}{8} + \cdot \cdot \cdot$

76. $9 + 6 + 4 + \frac{8}{3} + \cdot \cdot \cdot$

77. $\frac{1}{9} - \frac{1}{3} + 1 - 3 + \cdot \cdot \cdot$

78. $-\frac{125}{36} + \frac{25}{6} - 5 + 6 - \cdot \cdot \cdot$

Writing a Repeating Decimal as a Rational Number

In Exercises 79 and 80, find the rational number representation of the repeating decimal.

79. $0.\overline{36}$

80. $0.3\overline{18}$

Graphical Reasoning

In Exercises 81 and 82, use a graphing utility to graph the function. Identify the horizontal asymptote of the graph and determine its relationship to the sum.

81. $f(x) = 6\left[\frac{1 - (0.5)^x}{1 - (0.5)}\right], \quad \sum_{n=0}^{\infty} 6\left(\frac{1}{2}\right)^{n}$

82. $f(x) = 2\left[\frac{1 - (0.8)^x}{1 - (0.8)}\right], \quad \sum_{n=0}^{\infty} 2\left(\frac{4}{5}\right)^{n}$

83. **Depreciation** A tool and die company buys a machine for $175,000 and it depreciates at a rate of 30% per year. (In other words, at the end of each year the depreciated value is 70% of what it was at the beginning of the year.) Find the depreciated value of the machine after 5 full years.

84. **Population** The table shows the mid-year populations of Argentina (in millions) from 2009 through 2015. *(Source: U.S. Census Bureau)*

DATA — Spreadsheet at LarsonPrecalculus.com

Year	Population
2009	40.9
2010	41.3
2011	41.8
2012	42.2
2013	42.6
2014	43.0
2015	43.4

(a) Use the *exponential regression* feature of a graphing utility to find the nth term (a_n) of a geometric sequence that models the data. Let n represent the year, with $n = 9$ corresponding to 2009.

(b) Use the sequence from part (a) to describe the rate at which the population of Argentina is growing.

(c) Use the sequence from part (a) to predict the population of Argentina in 2025. The U.S. Census Bureau predicts the population of Argentina will be 47.2 million in 2025. How does this value compare with your prediction?

(d) Use the sequence from part (a) to predict when the population of Argentina will reach 50.0 million.

85. **Annuity** An investor deposits P dollars on the first day of each month in an account with an annual interest rate r, compounded monthly. The balance A after t years is

$$A = P\left(1 + \frac{r}{12}\right) + \cdot \cdot \cdot + P\left(1 + \frac{r}{12}\right)^{12t}.$$

Show that the balance is

$$A = P\left[\left(1 + \frac{r}{12}\right)^{12t} - 1\right]\left(1 + \frac{12}{r}\right).$$