12.2 Exercises

See CalcChat.com for tutorial help and worked-out solutions to odd-numbered exercises.

Vocabulary: Fill in the blanks.

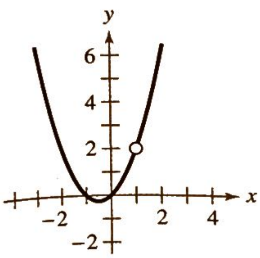
1. The fraction $\frac{0}{0}$ has no meaning as a real number and is called an ______ ______.
2. To evaluate the limit of a rational function that has common factors in its numerator and denominator when direct substitution fails, use the ______ ______ ______.
3. The limit $\lim_{x \to c^-} f(x) = L_1$ is an example of a ______ ______ ______.
4. The ______ ______ enables you to find the limit of a function that is squeezed between two functions, each of which has the same limit at a given x-value.

Skills and Applications

Using a Graph to Determine Limits

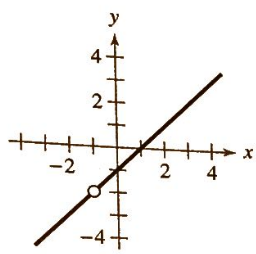
In Exercises 5 and 6, use the graph to find each limit visually. Then identify another function that agrees with the given function at all but one point.

5. $g(x) = \dfrac{x^3 - x}{x - 1}$



(a) $\lim_{x \to 1} g(x)$
(b) $\lim_{x \to -1} g(x)$
(c) $\lim_{x \to 0} g(x)$

6. $f(x) = \dfrac{x^2 - 1}{x + 1}$



(a) $\lim_{x \to 1} f(x)$
(b) $\lim_{x \to 2} f(x)$
(c) $\lim_{x \to -1} f(x)$

Finding a Limit

In Exercises 7–20, find the limit algebraically, if it exists. Use a graphing utility to verify your result graphically.

7. $\lim_{x \to 6} \dfrac{x^2 - 36}{x - 6}$
8. $\lim_{x \to 1} \dfrac{x^2 - 1}{x - 1}$
9. $\lim_{x \to 3} \dfrac{x - 3}{x^2 - x - 6}$
10. $\lim_{x \to -4} \dfrac{x + 4}{2x^2 + 9x + 4}$
11. $\lim_{x \to 0} \dfrac{\sqrt{x + 25} - 5}{x}$
12. $\lim_{x \to 0} \dfrac{\sqrt{x + 4} - 2}{x}$
13. $\lim_{x \to -3} \dfrac{\sqrt{x + 7} - 2}{x + 3}$
14. $\lim_{x \to 2} \dfrac{4 - \sqrt{18 - x}}{x - 2}$
15. $\lim_{x \to 0} \dfrac{\dfrac{1}{x + 1} - 1}{x}$
16. $\lim_{x \to 0} \dfrac{\dfrac{1}{x - 8} + \dfrac{1}{8}}{x}$
17. $\lim_{x \to 0} \dfrac{\sec x}{\tan x}$
18. $\lim_{x \to \pi} \dfrac{\csc x}{\cot x}$
19. $\lim_{x \to 0} \dfrac{\cos x - 1}{\sin x}$
20. $\lim_{x \to \pi/2} \dfrac{\cos x}{1 - \sin x}$

Approximating a Limit Graphically

In Exercises 21–30, use a graphing utility to graph the function and approximate the limit accurate to three decimal places.

21. $\lim_{x \to 0} \dfrac{e^{3x} - 1}{x}$
22. $\lim_{x \to 0} \dfrac{1 - e^{-x}}{x}$
23. $\lim_{x \to 0^+} (x \ln x)$
24. $\lim_{x \to 0^+} (x^2 \ln x)$
25. $\lim_{x \to 0} (1 - x)^{2/x}$
26. $\lim_{x \to 0} (1 + 2x)^{1/x}$
27. $\lim_{x \to 0} \dfrac{\sin 2x}{x}$
28. $\lim_{x \to 0} \dfrac{1 - \cos 2x}{x}$
29. $\lim_{x \to 1} \dfrac{1 - \sqrt[3]{x}}{1 - x}$
30. $\lim_{x \to 1} \dfrac{\sqrt[3]{x} - x}{x - 1}$

Using Different Methods

In Exercises 31–34, (a) graphically approximate the limit (if it exists) by using a graphing utility to graph the function, (b) numerically approximate the limit (if it exists) by using the *table* feature of the graphing utility to create a table, and (c) algebraically evaluate the limit (if it exists) by using the appropriate technique(s).

31. $\lim_{x \to 2} \dfrac{x^4 - 2x^2 - 8}{x^4 - 6x^2 + 8}$
32. $\lim_{x \to 2} \dfrac{x^4 - 1}{x^4 - 3x^2 - 4}$
33. $\lim_{x \to 16^+} \dfrac{4 - \sqrt{x}}{x - 16}$
34. $\lim_{x \to 0^-} \dfrac{\sqrt{x + 2} - \sqrt{2}}{x}$

Evaluating One-Sided Limits

In Exercises 35–38, graph the function. Find the limit (if it exists) by evaluating the corresponding one-sided limits.

35. $\lim_{x \to 1} \dfrac{|x - 1|}{x - 1}$
36. $\lim_{x \to -3} \dfrac{|x + 3|}{x}$
37. $\lim_{x \to 2} f(x)$, where $f(x) = \begin{cases} 3x - 2, & x < 2 \\ 8 - x^2, & x \geq 2 \end{cases}$
38. $\lim_{x \to 3} f(x)$, where $f(x) = \begin{cases} x^2 - 4, & x \leq 3 \\ x + 3, & x > 3 \end{cases}$