12.4 Exercises

Vocabulary: Fill in the blanks.

1. A _____ at _____ can help you solve the area problem in calculus.
2. For a rational function, when the degrees of the numerator and denominator are equal, the limit is the _____ of the leading coefficients.
3. A sequence that has a limit is said to _____.
4. A sequence that does not have a limit is said to _____.

Skills and Applications

Matching a Function with Its Graph In Exercises 5–8, match the function with its graph, using the asymptotes as aids. [The graphs are labeled (a), (b), (c), and (d).]

(a)

(b)

(c)

(d)
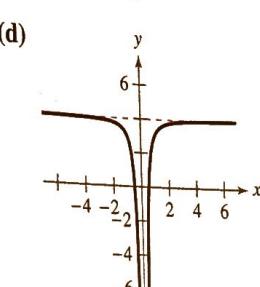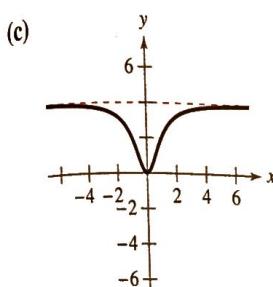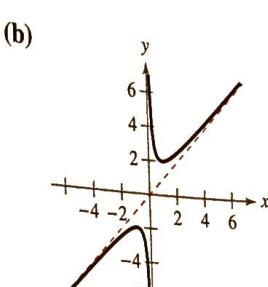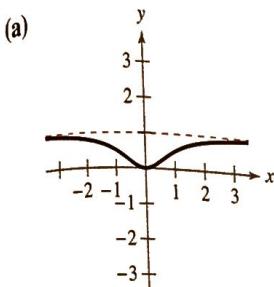
5. $f(x) = \dfrac{4x^2}{x^2 + 1}$

6. $f(x) = \dfrac{x^2}{x^2 + 1}$

7. $f(x) = 4 - \dfrac{1}{x^2}$

8. $f(x) = x + \dfrac{1}{x}$

Evaluating a Limit at Infinity In Exercises 9–28, find the limit, if it exists. If the limit does not exist, explain why. Use a graphing utility to verify your result graphically.

9. $\lim\limits_{x \to \infty} \left(2 + \dfrac{3}{x^2}\right)$

10. $\lim\limits_{x \to \infty} \left(4 - \dfrac{1}{x^2}\right)$

11. $\lim\limits_{x \to \infty} \dfrac{1 - x}{1 + x}$

12. $\lim\limits_{x \to \infty} \dfrac{1 + 5x}{1 - 4x}$

13. $\lim\limits_{x \to -\infty} \dfrac{3x^2 - 4}{1 - x^2}$

14. $\lim\limits_{x \to \infty} \dfrac{1 - 2x}{x + 2}$

15. $\lim\limits_{t \to -\infty} \dfrac{5x - 1}{3x^2 + 2}$

16. $\lim\limits_{x \to -\infty} \dfrac{4 + x}{2x^2 + 1}$

17. $\lim\limits_{t \to \infty} \dfrac{t^2}{t + 3}$

18. $\lim\limits_{y \to \infty} \dfrac{4y^4}{y^2 + 3}$

19. $\lim\limits_{t \to \infty} \dfrac{4t^2 - 2t + 1}{-3t^2 + 2t + 2}$

20. $\lim\limits_{x \to -\infty} \dfrac{2x^2 - 5x - 12}{1 - 6x - 8x^2}$

21. $\lim\limits_{x \to -\infty} \dfrac{-(x^2 + 3)}{(2 - x)^2}$

22. $\lim\limits_{x \to \infty} \dfrac{2x^2 - 6}{(x - 1)^2}$

23. $\lim\limits_{x \to \infty} \dfrac{x^3 + 3}{x^2}$

24. $\lim\limits_{x \to -\infty} \left(\dfrac{1}{2}x - \dfrac{4}{x^2}\right)$

25. $\lim\limits_{x \to -\infty} \left[\dfrac{x}{(x + 1)^2} - 4\right]$

26. $\lim\limits_{x \to \infty} \left[7 + \dfrac{2x^2}{(x + 3)^2}\right]$

27. $\lim\limits_{t \to \infty} \left(\dfrac{1}{3t^2} - \dfrac{5t}{t + 2}\right)$

28. $\lim\limits_{x \to \infty} \left[\dfrac{1}{2} + \dfrac{x^2}{(x - 3)^2}\right]$

Using Horizontal Asymptotes In Exercises 29–34, use a graphing utility to graph the function and estimate the horizontal asymptote and the limits at infinity for the function.

29. $y = \dfrac{3x}{1 - x}$

30. $y = \dfrac{x^2}{x^2 + 4}$

31. $y = \dfrac{5x}{1 - x^2}$

32. $y = \dfrac{2x + 1}{x^2 - 1}$

33. $y = 1 - \dfrac{3}{x^2}$

34. $y = 2 + \dfrac{1}{x}$

Estimating a Limit In Exercises 35–38, (a) complete the table and numerically estimate the limit as x approaches infinity for the function, and (b) use a graphing utility to graph the function and estimate the limit graphically.

x	10^0	10^1	10^2	10^3	10^4	10^5	10^6
$f(x)$							

35. $f(x) = x - \sqrt{x^2 + 2}$

36. $f(x) = 3x - \sqrt{9x^2 + 1}$

37. $f(x) = 3\left(2x - \sqrt{4x^2 + x}\right)$

38. $f(x) = 4\left(4x - \sqrt{16x^2 - x}\right)$